THE UPWARD SPEED OF AN AIR CURRENT NECESSARY TO SUSTAIN A HAILSTONE

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Of the several theories which have been put forward to explain the formation of the hailstone, one requires that the hailstone be sustained and lifted upwards by an ascending air current until near full size is attained. This has led Humphreys (**1**) and Simpson (**2**) to compute the upward air speed necessary to support a hailstone in order to arrive at an estimate of the speed with which air rushes upward in a thunderstorm. Since neither of these writers gives any details of the method of computation used, I shall supplement their work by developing a correct method of computation and also by taking into account the effect of turbulence in the air stream, an effect known to be large.

It is easy to show (**3**) by the application of the principle of dimensional homogeneity that, provided compressibility is neglected, the aerodynamical resistance of a body is expressible in the form

$$R=\tfrac{1}{2}\rho L^2V^2f\left(\frac{LV}{\nu}\right) \qquad (1)$$

which can be written in the dimensionless form

$$\frac{R}{\frac{1}{2}\rho L^2V^2}=f\left(\frac{LV}{\nu}\right) \qquad (2)$$

where

R = aerodynamical resistance
ρ = density of the air
L = some linear dimension of the body
V = relative air speed
$\nu=\frac{\mu}{\rho}$ = kinematic coefficient of viscosity
μ = coefficient of air viscosity.

The quantity $\frac{LV}{\nu}$, called Reynold's number, is the independent variable in equation (2) which is used to compute the results of wind-tunnel tests on resistance, or drag. That is, $\frac{R}{\frac{1}{2}\rho L^2V^2}$ is [1] plotted against $\frac{LV}{\nu}$ and the resulting curve [2] is the graphical representation of the function $f\left(\frac{LV}{\nu}\right)$. The form of this curve depends on the shape of the body and the turbulence of the air stream, and is the same for all bodies geometrically similar to each other provided the air stream always has the same turbulence. Since the hailstone will be assumed to be spherical, we shall confine our attention to the resistance of spheres. In this case L is replaced by D the sphere diameter, and equations (1) and (2) become

$$R=\tfrac{1}{2}\rho D^2V^2f\left(\frac{DV}{\nu}\right) \qquad (3)$$

$$\frac{R}{\frac{1}{2}\rho D^2V^2}=f\left(\frac{DV}{\nu}\right) \qquad (4)$$

Neglecting the effect of turbulence for the present, suppose the curve $f\left(\frac{DV}{\nu}\right)$ has been determined for a given sphere; then, since all spheres are geometrically similar, this same curve applies to a sphere of any size.

If a hailstone is to be sustained by a vertical air current, the upward drag of the air on the hailstone must be equal to its weight. Hence,

$$\tfrac{1}{2}\rho D^2V^2f\left(\frac{DV}{\nu}\right)=\tfrac{4}{3}\pi\frac{D^3}{8}d_w\rho_h \qquad (5)$$

where ρ_h is the specific gravity of the hailstone and d_w is the weight of water per unit volume. For a hailstone of given size and specific gravity and for given conditions of pressure and temperature of the atmosphere, ρ, D, ν, d_w, and ρ_h are constants so that equation (5) can be written

$$V^2f(K_1V)=K_2 \qquad (6)$$

where K_1 and K_2 are known constants and where $f(K_1V)$ is known for any given value of V. It is now required to solve equation (6) for V, for it is this value of V which is necessary to sustain the hailstone. It would be a simple matter to find this value of V if $f(K_1V)$ were constant, but experiment shows that the variation of $f(K_1V)$ with K_1V is far too large to be considered even approximately a constant. In this connection I might say that although reference **2** gives no information as to Simpson's method of computation, I have been able to determine by back-figuring from his results that $f(K_1V)$ was treated as a constant; his values of V must therefore be considered as uncertain. Since the function $f(K_1V)$ is not known analytically but only as a curve, equation (6) must be solved graphically. To do this, let

$$y=V^2f(K_1V)-K_2 \qquad (7)$$

so that the desired value of V is that which makes $y=o$. The procedure is to compute y for different values of V, plot y against V, and then pass a curve through these points; the required value of V is that at which this curve crosses the V axis. This method of computation is entirely general and can be used to compute the wind speed necessary to produce a given force on any body provided its resistance curve $f\left(\frac{LV}{\nu}\right)$ is known.

So much for the method of computation. Let us now consider the effect of turbulence in the air stream. The turbulence to be considered here is not the large scale turbulence usually thought of in connection with meteorology, but turbulence of a very much smaller scale such as is present in wind tunnel air streams and which is called fine-grained turbulence. In turbulence of this kind, small variations from the mean wind speed and direction occur many times in 1 second. Experiments show that the resistance of a sphere is particularly affected by the amount of turbulence of this type present in the air stream (**4**) and that the resistance curve of a sphere is

[1] The ratio $\frac{R}{\frac{1}{2}\rho L^2V^2}$ is usually denoted by C_D which is called the drag coefficient.

[2] Since $\frac{R}{\frac{1}{2}\rho L^2V^2}$ and $\frac{LV}{\nu}$ are both dimensionless, the curve of $f\left(\frac{LV}{\nu}\right)$ is independent of the size of the units used and is the same for all self-consistent systems of units.

different for different amounts of turbulence. Since this is the case, it is evident that the vertical air speed necessary to sustain a hailstone depends on the turbulence in the air current; in order to see how the necessary vertical speed is affected by turbulence, it is necessary to use the method outlined in connection with equation (7), using resistance curves $f\left(\frac{DV}{\nu}\right)$ determined under various conditions of turbulence. This has been done and the resulting values are given in table 1 for hailstones of various diameters.

The computations have been carried out on the assumption that the density of the air is three fifths that at sea level (the same density as used by Humphreys), corresponding to a height of about 14,000 feet. The coefficient of air viscosity is based on an assumed air temperature of 0° C.; the specific gravity of the hailstone is assumed to be 0.8. In the English gravitational system of units we have the following values:

$$\rho = \frac{3}{5} \times 0.00237 = 0.001422 \text{ slugs per cubic foot}$$

$$\mu = 0.3575 \times 10^{-6}$$

$$\text{Reynold's number} = \frac{DV}{\nu} = \frac{DV\rho}{\mu} = \frac{0.001422}{0.3575 \times 10^{-6}} DV$$

$$= 3978 DV$$

$$d_w = 62.4 \text{ pounds per cubic foot.}$$

As a sample computation let $D = 1.0 \text{ inch} = \frac{1}{12}$ foot. Equation (5) then becomes

$$\frac{1}{2} \times 0.001422 \times \frac{1}{(12)^2} \times V^2 f\left(3978 \times \frac{1}{12} \times V\right) = \frac{4\pi}{3}$$

$$\times \frac{1}{8 \times (12)^3} \times 62.4 \times 0.8,$$

or

$0.000004938\, V^2 f(331.5 V) = 0.01513$, which gives
$V^2 f(331.5 V) = 3065$ for equation (6),* and
$y = V^2 f(331.5 V) - 3065$ for equation (7).

Let

$$V = 60 \ \frac{\text{mi.}}{\text{hr.}} = 88.0 \frac{\text{ft.}}{\text{sec.}};$$

then

$$y = (88.0)^2 f(331.5 \times 88.0) - 3065,$$

or

$$y = 7744\, f(0.292 \times 10^5) - 3065.$$

Using the resistance curve of figure 12, page 11 of reference 5, it is found that $f(0.292 \times 10^5) = 0.36$. Hence, $y = 7744 \times 0.36 - 3065 = 2790 - 3065 = -275$, and by computing $y = V^2 f(331.5\ V) - 3065$ for a few more values of V it is found

that $y = o$ when $V = 63 \ \frac{\text{mi.}}{\text{hr.}}$

*It will be noted that from $V^2 f(331.5\ V) = 3065$ we can get the corresponding equation for a sphere of any diameter. Thus, for $D = 2.0$ inches equation (6) becomes $V^2 f(331.5 \times 2 \times V) = 3065 \times 2$, or $V^2 f(663.0\ V) = 6130$.

The values in column A of table 1 were obtained by using the resistance curve given in figure 12, page 11 of reference **5**. This curve was determined under conditions of excessive turbulence in the air stream (**6**) so that the values in column A correspond to a very turbulent condition. The values in column B were obtained by using the right hand resistance curve [3] in figure 7 (**7**). These measurements were made in an air stream having but little turbulence (**6**) so that the values in column B correspond to a condition of little turbulence. The values in column C were computed by using the resistance curve [4] in figure 16, page 14 of reference 5. These measurements were made by dropping spheres in the free air when there was extremely little turbulence; hence, the values in column C correspond to very little turbulence.

TABLE 1.—*Upward air speed necessary to sustain a spherical hailstone.*

Diameter of stone	A Much turbulence	B Little turbulence	C Very little turbulence
Inches	*Miles per hour*	*Miles per hour*	*Miles per hour*
1.0	63	65	[1] 54
2.0	206	91	[1] 78
3.0	244	112	[1] 101
4.0	258	264	[1] 123
5.0	272	--------	157

[1] Resistance curve interpolated. See footnote 4.

The large effect of turbulence is seen by comparing corresponding values of columns A and C. For instance, the upward air speed necessary to sustain a 2-inch hailstone may be anything from 78 $\frac{mi}{hr}$ to 206 $\frac{mi}{hr}$ depending on the state of turbulence in the ascening current of air. It is impossible to say definitely how much fine-grained turbulence might be present in ascending air currents which occur in thunderstorms. The few measurements which have been made show that under calm conditions the amount of such turbulence present in the free air is less than that in the smoothest of wind tunnels (**5**, p. 17). I understand that, in a paper not yet published, von Karman, of the California Institute of Technology, gives the results of measurements in the free air which confirm this. It might be argued that since the influence of the wind tunnel walls, honeycomb, and propeller is not present in the free air, the turbulence should vanish at a sufficient height above the surface of the earth. If this view is adopted, we might expect the necessary upward speeds to be even less than those in column C.

In order that its layer structure may be explained, the hailstone is usually assumed to rise and fall from one to several times before finally dropping to the earth. These vertical excursions are ordinarily attributed to variations in the upward air speed. It is interesting to note from table 1, however, that variations of the turbulence in the

[3] This curve is based on the equation $\frac{R}{\frac{1}{2}\rho\pi\frac{D^2}{4}V^2} = f\left(\frac{DV}{\nu}\right)$. Consequently for $D = 1.0$ inch equation (6) becomes $V^2 f(331.5 V) = 3902$.

[4] This resistance curve was extended to smaller values of $\frac{DV}{\nu}$ by using $\frac{R}{\frac{1}{2}\rho D^2 V^2} = 0.515$ at $\frac{DV}{\nu} = 0.0014 \times 10^5$. See bottom of p. 16, reference 5.

upward air current could also cause a succession of rises and falls of the hailstone, even though the upward air speed remained the same. At any rate, it is seen that variations in turbulence may be a factor in the vertical excursions of the hailstone.

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A BRILLIANT METEOR AND ITS CLOUD-LIKE TRAIL

By S. P. PETERSON

[Weather Bureau, Albuquerque, N.Mex., Mar. 24, 1933]

An unusually brilliant meteor was observed at 5:07 a.m on March 24, 1933, by Pilot C. W. Coyle, while flying a Transcontinental & Western Air Mail plane east from Albuquerque. At the time in question he was near Adrian, Tex., 235 miles east of Albuquerque, at a sea-level altitude of about 9,500 feet. At first it appeared as if a plane had suddenly turned on a landing light to the east of, and at an angular elevation of about 60° from, his position.

The meteor passed to the northward, seemingly at about his level, and looked like a ball of fire with pieces bursting from it, and left in its wake a great red trail tinged with blue. It disappeared to the westward and seemed to strike the earth, or disintegrate, northeast of Tucumcari, N.Mex.

The meteor also was seen by Pilot F. E.Williams when he was over Acomita, N.Mex., some 55 miles west of Albuquerque, flying a Transcontinental & Western Air Mail plane westward. Suddenly the sky was brilliantly illuminated, and on looking for the cause he saw the meteor behind him at an indefinite distance. It also was seen by several persons in Albuquerque. A very luminous cloud of bluish-green color, apparently developed by the meteoric dust, seemed to be suspended in the sky to the east-northeast of Albuquerque over the Sandia Mountains. This cloud remained visible until lost in the light of dawn. It seems that there was some electric development in the atmosphere along the passage of the meteor, as Pilot Coyle said that the radio beam that he was following at the time was cut out by a roar of static. There was much haziness over eastern New Mexico, southeastern Colorado, and the Texas Panhandle the latter part of the 24th, practically all the 25th, and locally in those sections early on the 26th. Whether this was owing to meteoric dust or to other causes is not known. The visibility at Dilia, N.Mex., and at Tucumcari, N.Mex., was reduced to one-half mile at the time of the greatest density of the haze. This meteor attracted the Nation-wide interest of scientists, who have made an extended search for its location.

The accompanying photograph of this meteor cloud was taken at 5:30 a.m. The camera was facing toward the east and the luminous cloud was apparently resting on the crest of the Sandia Mountains. There were a few scattering stratus and strato-cumulus clouds but these were still in the shadow of the earth, while the meteor cloud was in full sunshine, as shown in the picture.

When first seen by the photographer, the luminous cloud, looking like a magnesium flare, was midway between the top of the picture and the crest of the mountains, but it gradually settled, while he was preparing his camera, to the position in which it is here shown.

TROPICAL DISTURBANCES OF JULY 1933

By CHARLES L. MITCHELL

June 27–July 6.—This disturbance was first noted the evening of June 27, central in about latitude 9° north and longitude 59° west. It was the earliest known in that general area and also the only one in a record of nearly 50 years to pass south of the Island of Trinidad and over the northeastern corner of Venezuela. On the morning of June 28 the center was over the southwestern part of the Gulf of Paria. An Associated Press dispatch from Port of Spain, estimated that in the Island of Trinidad there were 13 deaths, 1,000 persons rendered homeless, about $3,000,000 property damaged, practically all in the southern part of the island.

Through the courtesy of the United States Chargé d'Affaires at Caracas, Venezuela, the following report has been received:

> *Hurricanes in eastern Venezuela.*—On June 28 a devastating hurricane swept through eastern Venezuela, the towns of Carúpano and Rio Caribe, on the mainland, and the island of Margarita suffering the most damage. Telephonic and telegraphic communications were cut for several days. Many business houses and private dwellings were destroyed, several small trading and fishing boats sunk and a number of lives lost. The losses from this hurricane alone are estimated at several millions of bolivars [1 bolivar= 19.3 cents]. During July there were several more hurricanes in the vicinity of Pedernales, at the mouth of the Orinoco, and along the river itself up as far as the Apure. However, most of them did not strike towns of any size.

During the next several days this disturbance moved first west-northwestward and later northwestward over the Caribbean Sea. It passed over extreme western Cuba the night of July 2–3, but did not cause much damage. By the morning of the 4th a strong area of high pressure, that spread southward from Hudson Bay over the eastern part of the United States, blocked the northward progress of this disturbance and deflected it toward the west. After moving westward until the evening of the 5th it turned southwestward and crossed the Mexican coast line about midway between Tampico and Brownsville, Tex., the evening of the 6th, where it caused several deaths and considerable property damage in the sparsely-settled coast region.

The usual twice-daily advisory warnings were issued in connection with this disturbance. Northeast storm warnings were ordered at noon of the 5th from Brownsville to Port O'Connor, Tex., and the warnings at Browns-